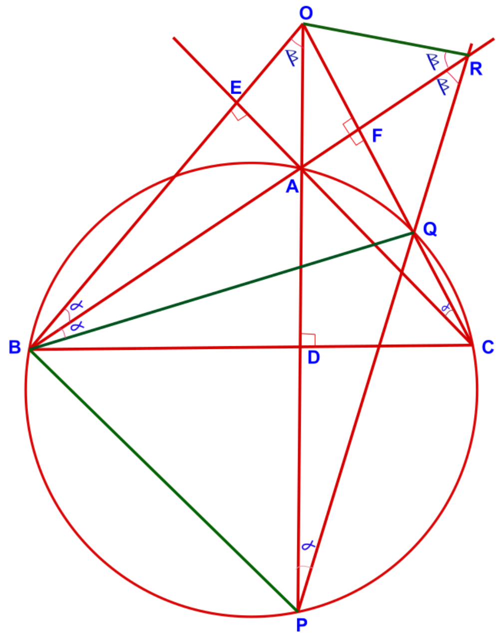
01.05.2023 CASH AWARD RIDER - AUTHOR'S SOLUTION



Given :

ΔABC is inscribed in the circle. Its altitudes AD, BE & CF are produced to meet at O, the orthocentre. AD produced meet the other side of the circle at P. CF cuts the circle at Q. BF and PQ are produced to meet at R.

To Prove : $OB^2 = AB \times BR$.

Construction : Join BP, BQ & OR

Solution:

$\angle QCA = \angle QPA = \angle QBA = \alpha$ (say) ---------------(1) (angles in the same segment)

$\angle BEC = \angle BFC = 90°$

$\therefore BEFC$ is concyclic

$\therefore \angle ABE = \angle ACF = \alpha$ -------------------------(2)

(1) & (2)→
$\angle QPA = \angle ABE = \alpha$
$\Rightarrow \angle RPO = \angle RBO = \alpha$

$\Rightarrow RPBO$ *is concyclic*

$\Rightarrow \angle BOP = \angle BRP = \beta$ (say) -------------------------(3)

(1) & (2) *also* $\Rightarrow \angle QBF = \angle OBF$

$\Rightarrow BR$ is the perpendicular bisector of OQ

$\Rightarrow \angle ORB = \angle QRB = \beta$ -----------------------(4)

(3) & (4) → $\angle ORB = \angle BOA = \beta$ ----------------(5)

In ΔORB & ΔAOB,

$\angle ORB = \angle AOB$ [From (5) above]

$\angle OBR = \angle OBA$ [same angle]

$\Delta ORB \sim \Delta AOB$

$$\frac{OR}{AO} = \frac{OB}{AB} = \frac{RB}{OB}$$

$\Rightarrow OB^2 = AB \times BR$. ----------------------Proved

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